Quick Quasi-TEM Analysis of Multiconductor Transmission Lines with Rectangular Cross Section

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Abstract—This paper presents an efficient and accurate procedure for computing the quasi-static matrix parameters ($[C]$, $[L]$, $[G]$, and $[R]$) of rectangular-shaped conductors embedded in a multilayered dielectric medium over an infinite ground plane. An additional top ground plane can also be considered. The problem is formulated in terms of the space-domain integral equation for the free-charge distribution on the slab conductor surfaces. The spatial Green's function is computed from its spectral counterpart using system identification techniques [Prony's method or matrix pencil method (MPM)]. The integral equation is solved by means of a Galerkin scheme employing entire domain basis functions. This results in a small matrix size. In addition, the quasi-analytical evaluation of the entries of the Galerkin matrix leads to a very efficient and accurate computer code. A detailed study on the convergence and accuracy of the method has been included.

Index Terms—Losses, microstrip, multiconductor transmission lines, quasi-TEM analysis, thick conductors.

I. INTRODUCTION

A LARGE amount of papers have been devoted to the analysis of planar transmission lines throughout the last three decades. Most of the published work assumes negligible metallization thickness. This approximation is good enough for many practical situations, and permits the simplification and efficient use of the analysis *ad hoc* mathematical techniques. However, during the last few years many authors have paid attention to the problem of accounting for the nonzero metallization thickness. Apart from the aim of increasing accuracy, this interest comes from the necessity of analyzing the electrical behavior of the relatively thick strips employed in monolithic microwave circuits and high-speed digital circuits. In these cases, metallization thicknesses and strip widths are in the same order of magnitude. Therefore, the former can no longer be neglected. The strip thickness must also be considered whenever tight edge coupling is present, even though relatively wide strips are under consideration. Moreover, the computation of ohmic conductor losses requires explicit accountability for the strip thickness.

A number of authors have analyzed the effects of the metallization thickness by using a full-wave analysis, but in this paper we are only interested in quasi-static approaches. This is because of its comparative simplicity, which makes it useful to develop quick microstrip solvers. Full-wave methods, except when applied to zero-thickness printed lines, usually demand a lot of computer time. Even when a quasi-TEM approach is used, accounting for strip-thickness effects precludes, in principle, the use of some very efficient analytical techniques developed for zero-thickness planar structures [1]–[3]. In principle, one might use some type of purely numerical approach, such as the finite-difference [4] or finite-element techniques [5]–[7]. However, even though significant improvements have been introduced in the formulation of those methods, they are more appropriate when dealing with complicated geometries which cannot be analyzed with lesser computer resource-demanding techniques. A hybrid numerical/analytical method—the method of lines—has been recently applied to the analysis of coupled microstrips with finite thickness [8]. Nevertheless, solutions based on integral-equation formulations seem to be well suited for most practical cases if the goal is to get high accuracy with low computational cost. Thus, arbitrary-section coupled conductors embedded in a layered medium have been analyzed in [9] by means of an integral-equation technique based on the free-space Green's function. This method was generalized in [10] to account for nonlayered dielectrics. Important numerical improvements on this technique have recently been reported in [11]. Other authors prefer to use a dielectric Green's function when dealing with layered dielectric substrates, since the number of unknowns is drastically reduced in this way [12]–[15]. The last five papers stressed the analytical preprocessing of the computations so as to enhance both accuracy and computational speed. Some other authors have reported different techniques to develop quick computational tools for the quasi-TEM analysis of particular microstrip structures with nonnegligible metallization thickness [16]–[19]. This paper fits into this research line.

In this paper, we propose a new method that combines the advantages of different formulations in order to build up a quick quasi-static computer solver for rectangular-shaped conductors embedded in a multilayered dielectric medium. The method is based on solving the space-domain integral equation for the free-charge distribution on the conducting slabs. The appropriate space-domain Green's function (the kernel) is conveniently obtained from the spectral one by means of system identification techniques (using the complex

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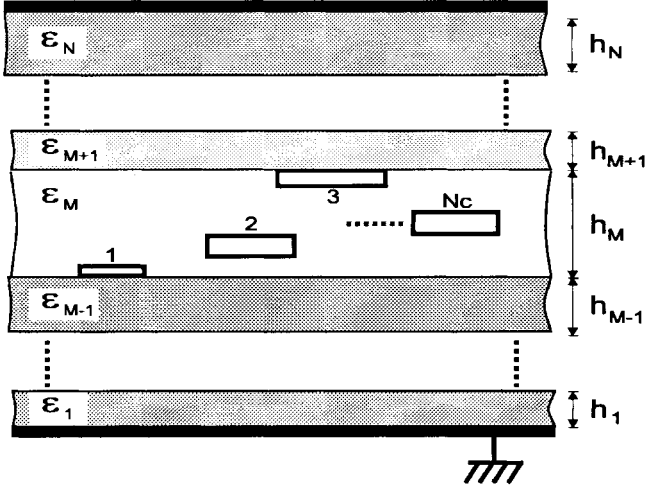
Optional ground plane

Fig. 1.   The multiconductor transmission line studied in this paper. It consists of N_c rectangular conducting slabs embedded in the Mth layer of a N-layers dielectric medium.

images concept [20]). Entire domain basis functions are used in a Galerkin scheme with the aim of keeping the size of the final Galerkin matrix small. In addition, the computation of the elements of this matrix is carried out in a very efficient way by using suitable numerical quadratures and closed-form integration. Putting together all these elements leads to an efficient and accurate computer code which is suitable for quick computations even on a PC platform.

II. STATEMENT OF THE PROBLEM

Consider a multiconductor transmission line such as the one shown in Fig. 1. An arbitrary number N_c of rectangular-shaped conductors are embedded inside the Mth dielectric layer of a multilayered (N layers) dielectric medium. A bottom ground plane is always present, while the top ground plane is optional. Our main purpose is to compute, in an efficient and accurate way, the per unit length complex capacitance matrix of the multiconductor system (i.e., the capacitance [C] and conductance [G] matrices). As is well known, the computation of the per unit length inductance matrix [L] reduces to the evaluation of [C] for the same structure without dielectrics. Finally, by using the Wheeler's incremental inductance rule, we can obtain the resistance matrix of the multiconductor system [R] from [L], as shown in [21].

As stated in the Section I, there exists a wide variety of techniques to compute the capacitance matrices of multiconductor transmission systems. We have chosen to solve the integral equation for the surface charge distribution on the rectangular conductors. This means that an appropriate Green's function accounting for the multiple boundaries and the bottom (and top) ground plates has to be computed. The spectral-domain version of such a Green's function can be easily obtained owing to the layered geometry of the dielectric region (see, for instance, the transverse transmission line (TTL) method reported in [22] or the method in [23]). If source and field points are inside the Mth layer, the spectral Green's function

can be written in the following form:

$$
\tilde{G}(\alpha|y,y') = \frac{1}{2\alpha\varepsilon_0\varepsilon_M(1 - \Gamma_L\Gamma_U e^{-2\alpha h_M})}[\Gamma_L e^{-\alpha(y+y')}
$$
$$
+ e^{-\alpha|y-y'|} + \Gamma_L\Gamma_U e^{\alpha(|y-y'|-2h_M)}
$$
$$
+ \Gamma_U e^{\alpha(y+y'-2h_M)}]. \tag{1}
$$

$\Gamma_{L,U}$ in (1) stand for the *reflection coefficients* seen from the lower (L) and upper (U) surfaces bounding the Mth layer, provided we are using the equivalent transmission-line model to compute the spectral Green's function [22]. Of course, $\Gamma_{L,U}$ are known closed-form functions of the Fourier variable α and of the permittivities and thicknesses of the dielectric layers below (L) or above (U) the Mth one. In order to recover the x-dependence of the space-domain Green's function, a Fourier transform inversion has to be carried out:

$$
G(x,x'|y,y') = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\alpha \tilde{G}(\alpha|y,y')e^{j\alpha(x-x')}. \tag{2}
$$

Apart from some particular cases, the inverse Fourier transform (2) cannot be performed in closed form. However, the α-dependent functions appearing in (1) as multiplicative factors of the exponential terms could be written as a finite sum of complex exponentials. In this way, we could apply the ideas reported in [20] so as to get a very close approximation to the space-domain Green's function. This point will be discussed in Section III. Once the Green's function is known, we can solve the integral equation for the free-charge distribution by using, for instance, the Galerkin method. In Section IV, we will give details on the type of basis functions used in this paper and on the techniques applied in order to speed up computations. From the charge on the conducting slabs, we obtain the capacitance matrix. If the electrical permittivities of the dielectric layers are complex (dielectric losses), the elements of this matrix will be complex. Their imaginary parts give the elements of [G]. On the other hand, [L] is the inverse of the capacitance matrix for the structure without dielectrics over c^2 (c is the speed of light in vacuo). Finally, under strong skin-effect operation, we can compute the per-unit-length resistance matrix [R], from [L] by using the extension to a multiconductor transmission-line system of the Wheeler's rule [21]. Therefore, with an accurate method to compute complex capacitance matrices, we will be able to obtain all the quasi-static matrix parameters characterizing our lossy multiconductor system (including the [R] matrix, provided the skin effect on metals is strong).

III. COMPUTATION OF THE SPACE-DOMAIN GREEN'S FUNCTION

Following the guidelines in [20], the space-domain Green's function can be usually expressed as a short summation of terms which can be easily computed from its spectral-domain version. The spectral Green's function is known in closed form for a layered structure. The basic equation underlying Chow's procedure is the following relationship between spectral and spatial functions:

$$
\mathcal{F}\left[-\frac{1}{2\pi}\ln((x-x')^2 + u^2)\right] = \frac{e^{-\alpha u}}{\alpha} \tag{3}
$$

where $\mathcal{F}$ stands for Fourier transform and u is a generalized *complex distance*. If we can write (1) as a sum of terms such as the ones on the right-hand side (RHS) of (3), it is clear that we will have the space-domain Green's function of our problem. This can be done almost in a straightforward way by expanding the terms by multiplying the exponentials in (1) as a sum of complex exponentials. The arguments of the exponentials and the coefficients of the expansion can be computed by using spectral estimation methods. In this paper, we have used and compared the Prony's method employed in [20] and the MPM, as reported in [24].

A caution should be exerted when we are considering structures having a top ground plane in addition to the bottom one. In such a case, the spectral functions to be approximated are singular at the point $\alpha = 0$. This behavior comes from the following factor:

$$\frac{1}{1 - \Gamma_L \Gamma_U e^{-2\alpha h_M}} \approx \frac{1}{2\alpha H[1 + \alpha(2d - H)]}, \qquad \alpha \to 0 \tag{4}$$

where $H = \varepsilon_M \sum_{i=1}^{N} (h_i/\varepsilon_i)$ and $d = \varepsilon_M \sum_{i=1}^{M-1} (h_i/\varepsilon_i)$. A homogeneous structure with relative permittivity ε_M having two ground planes separated by a distance H has a spectral Green's function that can be expressed as follows:

$$\tilde{G}_h(\alpha|y, y') = \frac{1}{2\alpha\varepsilon_0\varepsilon_M(e^{-2\alpha H} - 1)} [e^{-\alpha(y+y')}$$
$$- e^{-\alpha|y-y'|} - e^{\alpha(|y-y'|-2H)} + e^{\alpha(y+y'-2H)}]. \tag{5}$$

The Green's function in (5) and the original one have the same behavior around the $\alpha = 0$ point. In addition, the space-domain Green's function for the homogeneous structure $G_h(x, y|x', y')$ is known in closed form:

$$G_h(x, y|x', y')$$
$$= \frac{1}{4\pi\varepsilon_0\varepsilon_M}$$
$$\times \ln\left\{\frac{\cosh[\pi|x - x'|/H] - \cos[\pi(y + y')/H]}{\cosh[\pi|x - x'|/H] - \cos[\pi(y - y')/H]}\right\}. \tag{6}$$

Expression (6) can be considered as the first contribution to the complete Green's function of the layered structure. The additional terms are expressed in the spectral domain as the difference between (1) and (5). The terms of this spectral function have no singularities at $\alpha = 0$ and can now be treated without problems.

In brief and after some algebra, the spectral-domain Green's function for the layered structure can be written in the following useful form:

$$\tilde{G}(\alpha|y, y')$$
$$= \frac{1}{2\alpha\varepsilon_0\varepsilon_M}\{e^{-\alpha|y-y'|} + K e^{-\alpha(y+y')} + A_1 e^{-\alpha(y+y')}$$
$$+ A_2(e^{-\alpha|y-y'|} + e^{\alpha|y-y'|}) + A_3 e^{\alpha(y+y')}\}$$
$$+ \delta\left[\tilde{G}_h(\alpha|y, y') - \frac{1}{2\alpha\varepsilon_0\varepsilon_M} \cdot e^{-\alpha|y-y'|}\right] \tag{7}$$

where

$$A_1(\alpha) = \frac{\Gamma_L}{1 - \Gamma_L \Gamma_U e^{-2\alpha h_M}} - K - \delta\frac{e^{-2\alpha d}}{e^{-2\alpha H} - 1} \tag{8a}$$

$$A_2(\alpha) = \frac{\Gamma_L \Gamma_U e^{-2\alpha h_M}}{1 - \Gamma_L \Gamma_U e^{-2\alpha h_M}} + \delta\frac{e^{-2\alpha H}}{e^{-2\alpha H} - 1} \tag{8b}$$

$$A_3(\alpha) = \frac{\Gamma_U e^{-2\alpha h_M}}{1 - \Gamma_L \Gamma_U e^{-2\alpha h_M}} - \delta\frac{e^{-2\alpha(H+d)}}{e^{-2\alpha H} - 1} \tag{8c}$$

being

$$K = \frac{\varepsilon_M - \varepsilon_{M-1}}{\varepsilon_M + \varepsilon_{M-1}}.$$

In all the above expressions $\delta = 1$ in the presence of a top ground plane, and $\delta = 0$ if there is no top ground plane.

The first two terms at the RHS in (7) correspond to the source point and the first real image. The spectral functions $A_i(\alpha) \{i = 1, 2, 3\}$ can be expanded as sums of complex exponentials. These terms can be viewed as the contributions of certain *complex images*, following the terminology in [20]. Usually just a few of these *complex images* are enough to get a very accurate representation of the Green's function. Therefore, after approximating the A_i coefficients in (8) as short series of complex exponential functions with arguments $(-\alpha b_n)$ and amplitudes B_n, we obtain from (3) and (7) the following space-domain Green's function:

$$G(x, y|x', y')$$
$$= -\frac{1}{4\pi\varepsilon_0\varepsilon_M}\left\{\ln[(x - x')^2 + (y - y')^2] + \frac{\varepsilon_M - \varepsilon_{M-1}}{\varepsilon_M + \varepsilon_{M+1}}\right.$$
$$\cdot \ln[(x - x')^2 + (y + y')^2]$$
$$+ \sum_{n=1}^{N_{ci}} B_n \ln[(x - x')^2 + (y \pm y' \pm b_n)^2]\right\}$$
$$+ \delta\left\{G_h(x, y|x', y') + \frac{1}{4\pi\varepsilon_0\varepsilon_M}\right.$$
$$\left. \cdot \ln[(x - x')^2 + (y - y')^2]\right\} \tag{9}$$

where N_{ci} is the total number of employed complex images. Once again, the term affected by the factor δ in (9) is present only if we have a top ground plane. This term does not present a logarithmic singularity in the definition domain and, therefore, will not lead later to integration problems. Nevertheless, in the computer implementation of the code, we have extracted out not only the singular term, but also the first two real images (reflections on the top and bottom ground planes) in order to avoid lengthy integrations when the source and field points are very close to the ground plates.

In order to get some insight about the features of the approximation we are using, we include in this section some numerical examples. Thus, Fig. 2 shows the relative error when a typical spectral function—such as the ones found in our problem—is approximated by an increasing number of complex images. In this example, we have approximated
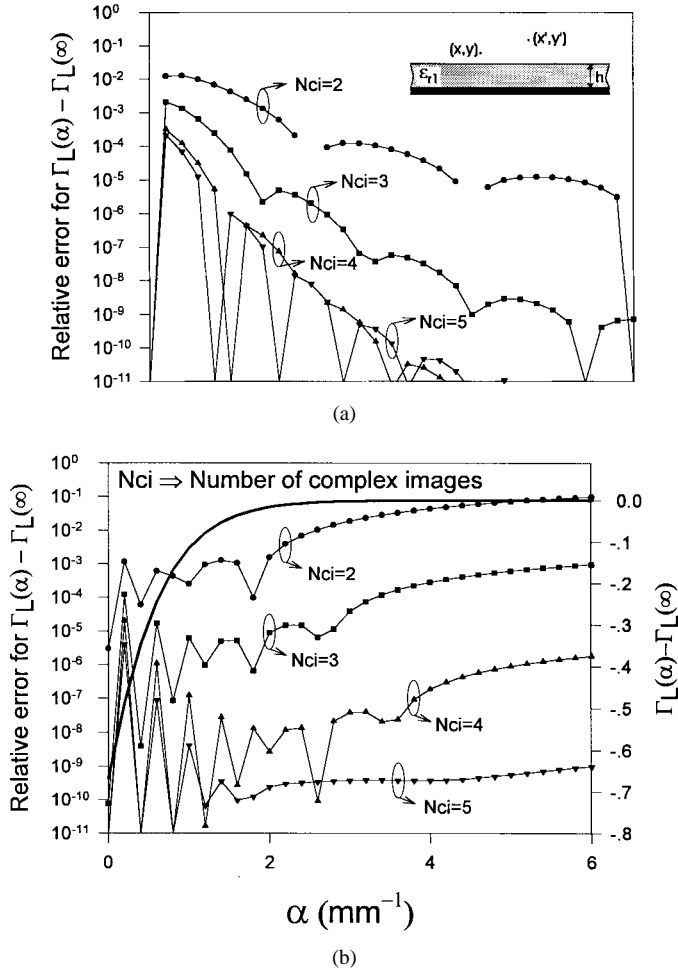
(a)

(b)

Fig. 2. Relative error in the approximation of a typical spectral function in our problem by using (a) Prony's method, and (b) MPM. We present curves for 2–5 complex images. $\Gamma_L(\alpha)$ corresponds to a single dielectric layer ($\varepsilon_r = 2, h_1 = 1$ mm) on a perfect ground plane.

$\Gamma_L(\alpha) - \Gamma_L(\infty)$, which is the sole function to be approximated when dealing with the standard microstrip structure. The complex images have been found by applying Prony's method [Fig. 2(a)] and the MPM [Fig. 2(b)]. From these figures we conclude that no more than four or five images are necessary to get a very good representation of the original Green's function in the spectral domain and, therefore, in the space domain. Fig. 2(b) also includes the function to be approximated. Notice that although the Prony approximation is more accurate when α increases, the use of the MPM seems to be advisable because this approach yields better results in the region where the approximated function is meaningfully different from zero. This feature has been confirmed for a large number of numerical examples. In brief, we conclude after a lot of numerical tests that just a few images computed with the MPM will ensure a high-quality approximation to the required space-domain Green's function.

IV. APPLICATION OF THE GALERKIN METHOD

In Section III, we have described a simple method to obtain an approximate closed-form expression for the space-domain Green's function. Now we have to solve the integral equation for the surface free-charge distribution on the rectangular conductors by using the Galerkin method. It is well known that the efficiency of this technique depends to a large extent on the suitability of the chosen basis functions. In our particular case we should use functions accounting for the singular behavior at the metallic corners. Nevertheless, we have chosen a different criterion. We are more interested in using functions leading to closed-form formulas for the elements of the Galerkin matrix in order to speed up the filling of such a matrix. But, in addition, the number of functions needed to get a given accuracy should be kept as low as possible. These two goals can be achieved by using each of the faces of the rectangular conductors or the basis functions usually employed to approximate the charge distribution on zero thickness strips: first-kind Chebyshev polynomials weighed by the Maxwell distribution. In this way, for a given conducting face of width w we write

$$\sigma(t) \approx \frac{2}{\pi w} \sum_{n=0}^{n_f} C_n \frac{T_n\left(\dfrac{t-t_0}{w/2}\right)}{\sqrt{1 - \left(\dfrac{t-t_0}{w/2}\right)^2}} \qquad (10)$$

where t stands for the x or y variable (for horizontal and vertical conducting faces, respectively), t_0 is the middle point of the interval where the charge density is approximated, and n_f is the number of retained basis functions on that face.

An important point is that the functions in (10) only partially account for the singularities at the corners. However, they allow us a quick filling of the Galerkin matrix. This is because some of the required integrations can be performed in closed form. When we use the Green's function in the form given in (9) and the basis functions in (10), we have to carry out the following integrations:

$$\int_{t_0-w/2}^{t_0+w/2} \ln\left[(x-x')^2+(y\pm y'\pm b_n)^2\right] \cdot \frac{T_p\left(\dfrac{t'-t_0}{w/2}\right)}{\sqrt{1-\left(\dfrac{t'-t_0}{w/2}\right)^2}}\,dt' \tag{11}$$

where t' stands for x' or y'. The integrals in (11) can be written in terms of the following one:

$$\int_{-1}^{1} \ln(z_1 + tz_2 + jz_3)\frac{T_p(t)}{\sqrt{1-t^2}}\,dt = f(z_1, z_2, z_3). \tag{12}$$

This integral has a closed form and can be expressed as follows:

Case a) $(p > 0)$:

$$f(z_1, z_2, z_3) = -\frac{\pi}{p}[\xi(z_1, z_2, z_3)]^p \tag{13a}$$

Case b) $(p = 0)$:

$$f(z_1, z_2, z_3) = \pi \ln\left[\frac{-z_2}{2\xi(z_1, z_2, z_3)}\right] \tag{13b}$$

where

$$\xi(z_1, z_2, z_3) = -\frac{z_1 + jz_3}{z_2} \pm \sqrt{\left(\frac{z_1 + jz_3}{z_2}\right)^2 - 1}.$$

(The sign before the square root is chosen in such a way that $|\xi| < 1$).

We must mention here that the real part of the integral in (12) was reported by Fikioris *et al.* in [1], and that we have also essentially used the same technique to get the imaginary part of (12).

The remaining integrations (inner products with the test functions) can now be carried out by using low-order Gauss–Chebyshev quadratures. If a top ground plane is present we have to still account for the contribution of the δ term in (9). This is easily done performing a double low-order Gauss–Chebyshev quadrature, owing to the mathematical nature of the integrand. The final result is that the Galerkin matrix is filled with low computational effort. This could also be done for simple subsectional pulse or triangular basis functions. The advantage of using the functions in (10) is that we do not need too many of them to get very accurate results, as will be shown in the Section V. Therefore, the Galerkin-matrix size will be small and the overall computation time will be low.

V. NUMERICAL RESULTS

As a first step in the analysis of the numerical behavior of the proposed technique, we have identified the different factors affecting the accuracy of the final results. In Section III, we said some words about the computation of the space-domain Green's function. Since this function is numerically computed by means of an approximate method, its values will be affected by a certain error. However, this error can be systematically reduced by increasing the number of complex images. In fact, just a few of these images ensure a relative error well below one part in 10^5 in the whole range of interest. Another source of error of numerical type can be found in the evaluation of the numerical quadratures needed to compute some integrals. After a lot of numerical experiments, we concluded that very good results will be obtained by using a number of quadrature points exceeding in two the order of the higher order Chebyshev polynomial used in the basis-function's expansion. In the case of inner products corresponding to functions defined on touching strip segments, this number should be increased to 10. With this choice, we do not detect errors associated with erroneous computation of definite integrals.

However, the main factor affecting the accuracy of the final results is the number of basis functions retained in the expansion of the free-charge distribution. We are interested in getting good enough results with few basis functions so as to keep the size of the Galerkin matrix small. Obviously, our computer code permits us to consider the case of zero-thickness strips as a particular case. We have made comparisons of the results provided by our code with other practically exact results reported in the literature [1]–[3]. Our

TABLE I
C_{11} (+) AND C_{12} (−) (pF/m) FOR THE STRUCTURE IN THE FIGURE AGAINST THE NUMBER OF BASIS FUNCTIONS ON VERTICAL (N_t) AND HORIZONTAL (N_w) CONDUCTOR FACES. $w = 1$, $s = t = 0.2$, $h_1 = 3$, $h_2 = 2$ (mm). $\varepsilon_{r1} = 2.5$, $\varepsilon_{r2} = 10$

		N_t				
		1	2	3	4	5
N_w	1	210.69	211.55	211.84	211.92	211.97
		-102.46	-103.17	-103.36	-103.43	-103.47
	2	216.70	216.85	217.11	217.12	217.14
		-107.60	-107.75	-107.93	-107.94	-107.96
	3	216.98	217.05	217.37	217.37	217.40
		-107.66	-107.72	-108.01	-108.02	-108.04
	4	217.01	217.06	217.38	217.39	217.42
		-107.68	-107.73	-108.03	-108.03	-108.06
	5	217.10	217.14	217.40	217.40	217.43
		-107.76	-107.79	-108.04	-108.04	-108.06
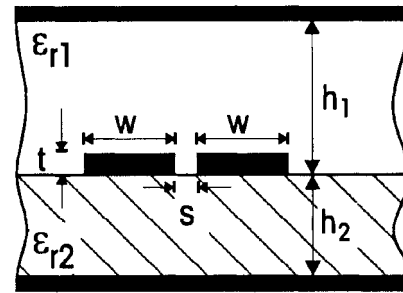
program also yields virtually exact results because for the zero thickness case, it is a quasi-analytical microstrip solver (such as the methods reported in the cited papers), with extremely low central processing unit (CPU) time consumption. This excellent performance is related to the nature of the used basis functions—which are especially suitable for zero thickness strips—and to the analytical treatment of the computation of the Galerkin matrix entries. Therefore, as a subproduct of our analysis, we have an extremely efficient code for the analysis of zero thickness coupled strips available. Nevertheless, in the context of this paper, we are more interested in the case of nonzero thickness strips. Therefore, we show in Table I the convergence of the capacitance coefficients for a pair of thick coupled microstrips. From Table I, we can see that convergence is not so good as the one we achieved in the analysis of zero-thickness coupled strips [2]. The reason for this is that the employed basis functions *do not* exactly fit the corner behavior. However, these functions are very good from a practical point of view: two basis functions per strip side yield results with an accuracy better than 0.3%. Of course, the number of basis functions has to be increased when the geometry is more critical (for example, when coupling between the strips is extremely strong or in other geometrically complicated situations). Nevertheless, we usually obtain very good results with just a few basis functions per strip side. Getting similar accuracy with subsectional-type basis functions would require many more basis functions, therefore increasing the CPU time.

After analyzing the convergence of the method, it is interesting to check the typical attainable accuracy using some

TABLE II
Z_0 OF A SYMMETRICAL SHIELDED SLAB LINE $N_{w,t}$ IS THE
NUMBER OF BASIS FUNCTIONS W = SLAB WIDTH. t = SLAB
THICKNESS b = SEPARATION BETWEEN GROUND PLATES

$W/(b-t)$	t/b	Exact [25]	$N_{w,t} = 8$	$N_{w,t} = 3$	$N_{w,t} = 1$
0.1	0.0	194.226	194.226	194.226	194.226
0.1	0.1	145.665	145.665	145.676	146.137
0.1	0.5	83.262	83.261	83.270	83.413
0.1	0.9	43.079	43.074	43.085	43.679
0.2	0.0	153.029	153.029	153.029	153.030
0.2	0.1	123.293	123.293	123.304	123.747
0.2	0.5	75.928	75.928	75.937	76.165
0.2	0.9	41.054	41.050	41.063	41.529

TABLE III
NORMALIZED RESISTANCE OF RECTANGULAR SLAB OVER GROUND PLANE AS A
FUNCTION OF THE NUMBER OF BASIS FUNCTIONS (nf) ON EACH STRIP SIDE
AND OF THE $\Delta\xi$ INCREMENT USED IN THE NUMERICAL DERIVATION SLAB WIDTH
= $2a$, SLAB THICKNESS = a, DISTANCE TO GROUND PLANE = a

	aR/R_s			
nf	$\Delta\xi = a \times 10^{-1}$	$a \times 10^{-2}$	$a \times 10^{-3}$	$a \times 10^{-4}$
1	0.23541	0.23394	0.23392	0.23392
2	0.23542	0.23395	0.23393	0.23393
3	0.23757	0.23607	0.23605	0.23605
4	0.23758	0.23608	0.23607	0.23607
5	0.23758	0.23608	0.23606	0.23606

appropriate benchmark. We can use as a benchmark the rectangular slab symmetrically placed between two infinite ground planes, since for this structure there exists an analytical solution. Table II shows the comparison between exact solutions taken from [25] (conformal mapping approach) and numerical values computed with our method. It is clear from this table that our method can provide very high accuracy if we use enough basis functions. However, even using just one function on each side of the slab, we get results within a 0.4% error except for the worst case ($t/b = 0.9$), where the error is around 1.5%. In this case, using just one more function on the long sides of the slab results in a drastic improvement of the accuracy (less than 0.02% error). Other comparisons have been made with some exact results reported in [26] with similar conclusions. Convergence to the correct value has then been demonstrated. Note that this point is important since our basis functions do not exhibit the rigorously correct edge behavior.

In [19], the author presents a technique based on spectral-domain analysis (SDA) to compute the quasi-static parameters of boxed coupled strips of arbitrary thickness. We have made comparisons with the results reported in [19] and we have found very good agreement. In particular, we have computed the modal parameters of two very tightly coupled strips, which cannot be efficiently treated by using the simple multistrip model reported in [15] (as was claimed in [19]). Our results are very accurate for this case even using only two basis functions on the faced sides of the strips and one on the remaining sides (0.2% error). This results in CPU times much shorter than the ones reported in [19] (ours is below 1 s on a 66-MHz pentium-based PC).

Our program can be used to calculate the resistance matrix $[R]$ of the multiconductor system just applying the Wheeler's rule, such as discussed in [21]:

$$[R] = \frac{R_s}{\mu_0} \frac{\partial [L]}{\partial \xi}$$

where R_s is the surface resistance of the metal, $[L]$ is the inductance matrix, and ξ is the coordinate variable normal to the conductor surface. The derivative is numerically computed, so we only need to accurately compute $[L]$. Table III shows the normalized resistance for a rectangular slab on a ground
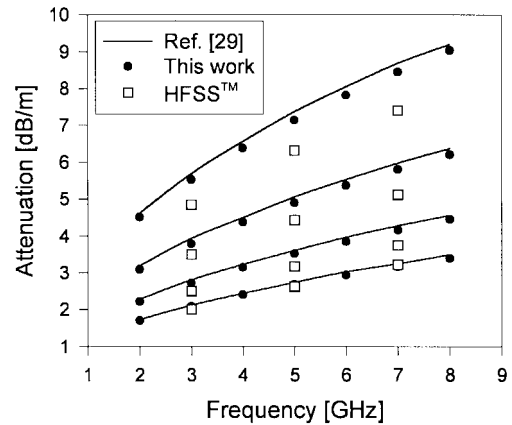

Fig. 3. Attenuation factors for the fundamental quasi-TEM modes of the four microstrips system analyzed in [27]. Strip widths: $w_1 = w_4 = 0.6$ mm, $w_2 = w_3 = 0.3$ mm, separation between strips: $s_1 = s_3 = 0.3$ mm, $s_2 = 0.2$ mm, strip thickness: $t = 0.01$ mm, substrate thickness: $h = 0.635$ mm, height to cover: $d = 6.0$ mm, $\varepsilon = 9.8\varepsilon_0$, $\sigma = 5.1 \cdot 10^7$ S/m

plane as a function of the number of basis functions used on each strip side. The result is close to the one reported in [13] ($R/R_s = 0.23753$). We have included several columns to show how the $\Delta\xi$ increment used to numerically perform the derivation affects the final result. We can typically choose $\Delta\xi$ around 10^{-3} times the smallest dimension of the conductor. The result remains stable for lower values of $\Delta\xi$. Note that the derivative is very accurately computed even though the original function is affected by a certain error, since this is a systematic error that affects in the same magnitude the two values of the function required to numerically perform the derivation. Recently, Gentili et al. [27] have reported a technique to compute the modal attenuation factors (α_k $k = 1, \cdots, N_c$) for a multiconductor line. The values of the α_k parameters can be readily obtained from the characteristic matrices computed in this paper. This way, we have compared our results with those reported in [27], with very good agreement. For instance, we show in Fig. 3 the four modal attenuation factors corresponding to the fundamental modes supported by a four-coupled-strip lossy system. Our results are very close to the ones computed by the method proposed by Gentili et al. (slight differences could be attributed to the side walls considered in that paper), but some important discrepancies have been found when comparing the results computed by means of a finite-element-based code.

TABLE IV
COEFFICIENTS OF [C], [L], AND [R] FOR THE FIVE CONDUCTORS' STRUCTURE IN THE FIGURE. $w = 3$, $s = 2$, $h = 1$, $t = 1$ (mm), $\varepsilon_{r1} = 2$, $\varepsilon_{r2} = 1$.

i	j	C_{ij}(pF/m)	L_{ij}(μH/m)	R/R_s
1	1	93.668	19.781×10^{-2}	0.1885
1	2	-8.453	2.946×10^{-2}	0.1347×10^{-1}
1	3	-0.809	0.735×10^{-2}	0.4600×10^{-2}
1	4	-0.345	0.284×10^{-2}	0.2657×10^{-2}
1	5	-0.215	0.152×10^{-2}	0.1605×10^{-2}
2	2	95.329	19.471×10^{-2}	0.1939
2	3	-8.318	2.883×10^{-2}	0.1459×10^{-1}
2	4	-0.758	0.717×10^{-2}	0.4879×10^{-2}
3	3	95.341	19.458×10^{-2}	0.1941
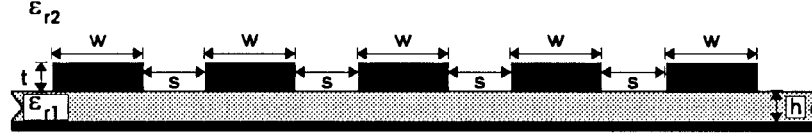
Finally, Table IV shows the elements of [C] and [L] for a five-coupled-strip system. All the figures in Table IV are correct. We have used 15 basis functions on each strip side, i.e., a total of 300 basis functions, to ensure the accuracy of these results. This results in a computation time of 75 s on a Pentium PC/66 MHz. However, accuracy better than 0.2% for all the matrix elements is achieved by using five functions and 13 s of CPU time. If an error in the order of 1% can be tolerated, we only have to use two functions per strip side. We have completed Table IV, including the normalized resistance matrix elements.

VI. CONCLUSIONS

This paper describes an efficient and accurate technique to compute the quasi-static matrix parameters ([C], [L], [R], [G]) of a system of rectangular cross-section coupled conductors embedded in a layered dielectric medium. Accuracy and numerical efficiency are achieved by means of two main issues. First, we use entire domain basis functions to approximate the free surface-charge distribution on the strips. These functions allow us to keep the Galerkin matrix size small when compared with typical matrix size associated with the use of subsectional functions. Second, we get quasi-analytical evaluation of the integrals defining the Galerkin-matrix entries. This has been done by taking advantage of the use of the complex image concept and the mathematical properties of the employed basis functions. Any spectral estimation technique can be used to compute the complex images, but the MPM seems to perform better than Prony's method. The final product is an accurate and quick computer code that permits one to analyze under quasi-TEM assumption a variety of transmission lines consisting of coupled rectangular conducting slabs. The performance of the code has been exhaustively checked by making convergence tests and comparisons with other techniques, some of which have been included in Section V. Note that although in this paper it has been assumed that all the strips were embedded in the same dielectric region, a more general situation can be treated by applying the same general ideas reported here.

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